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# Aeroelastic Panel Optimization with Aerodynamic Damping

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## Nomenclature

$a$ = panel length
$D$ = panel bending stiffness
$g$ = aerodynamic damping parameter, $\rho V a^2(M^2-2)/[\pi^2(m_o D_o)^{1/2}(M^2-1)^{3/2}]$
$m$ = panel mass per unit length
$M$ = freestream Mach number
$p$ = nondimensional slope, $w'$
$q$ = nondimensional bending moment, $t^3(x)w''$
$r$ = nondimensional shear, $[t^3(x)w'']'$
$R_x$ = inplane loading parameter, $N_x a^2/D_o$; $N_x$ = tensile stress
$t$ = nondimensional thickness ratio, $T/T_o$; $T$ = panel thickness
$t_{\min}$ = minimum allowable thickness ratio
$u$ = unconstrained control function; see Eq. (5)
$V$ = freestream air speed
$w$ = panel deflection divided by panel length $a$
$x$ = distance along panel divided by panel length $a$
$\alpha^2$ = ratio of fundamental flutter frequency to fundamental frequency of free vibration
$\lambda$ = critical aerodynamic flutter parameter, $\rho V^2 a^3/[D_o(M^2-1)^{1/2}]$
$\rho$ = freestream air density

*Subscripts*
$I$ = imaginary part
$R$ = real part
$o$ = uniform reference panel value

## Introduction and Problem Statement

SEVERAL authors[1–7] have investigated a class of one-dimensional, minimum-weight, supersonic panel design problems in which the flutter speed is specified. Optimal thickness distributions are available for both homogeneous and sandwich panels, and for various levels of constant inplane loading and specified minimum thickness. The purpose of this Note is to present some numerical solutions for the case in which aerodynamic damping is included in the problem formulation.

Under the assumptions of linear elastic bending and linearized aerodynamic strip theory for high Mach number supersonic flow,[8] the nondimensional equation of equilibrium for assumed simple harmonic motion of an infinite-span solid panel is

$$[t^3(x)w'']'' + R_x w'' + \lambda_o w' + i\alpha^2\pi^4 g_o w - (\alpha\pi)^4 t(x) w = 0 \qquad (1)$$

The fourth term of Eq. (1) represents the damping due to aerodynamic forces. Since this term is imaginary, Eq. (1) is complex and may be regarded as two 4th-order dynamic systems coupled by the damping term. For a simply supported solid panel, the problem is to find that thickness ratio distribution $t(x)$, $0 \leqq x \leqq 1$, and fundamental frequency parameter $\alpha$ which minimize the mass ratio $\int_0^1 t(x)\,dx$ subject to the differential equations

$$\begin{aligned} w'_R &= p_R; \quad p'_R = q_R/t^3(x); \quad q'_R = r_R \\ r'_R &= -R_x q_R/t^3(x) - \lambda_o p_R + \alpha^2\pi^4 g_o w_I + (\alpha\pi)^4 t(x) w_R \\ w'_I &= p_I; \quad p'_I = q_I/t^3(x); \quad q'_I = r_I \\ r'_I &= -R_x q_I/t^3(x) - \lambda_o p_I - \alpha^2\pi^4 g_o w_R + (\alpha\pi)^4 t(x) w_I \end{aligned} \qquad (2)$$

with boundary conditions

$$\begin{aligned} w_R(0) &= q_R(0) = w_I(0) = q_I(0) = 0 \\ r_R(0) &= 1 \\ w_R(1) &= q_R(1) = w_I(1) = q_I(1) = 0 \end{aligned} \qquad (3)$$

and subject to the inequality constraint

$$t(x) \geqq t_{\min}, \quad 0 < t_{\min} < 1, \quad 0 \leqq x \leqq 1 \qquad (4)$$

The inplane loading parameter $R_x$ and the aerodynamic damping parameter $g_o$ are regarded as specified constants. The value of the aerodynamic parameter $\lambda_o$ is held fixed at its critical value for flutter onset which in turn depends on $R_x$ and $g_o$.[8] Since the dependent variables of Eq. (2) can be arbitrarily scaled without altering the necessary conditions for optimality,[7] an advantageous choice of scaling can be made by setting $r_R(0) = 1$ as shown in Eq. (3).

Plaut[9] has applied a two-term Ritz procedure to the related problem of maximizing the critical aerodynamic parameter $\lambda_o$ while maintaining a given total panel mass. His simplified analysis, which is apparently the only available result regarding damping for these panel optimization problems, indicates that the addition of damping can have a significant effect. However, it is difficult to predict general trends for the minimum-weight problem from his results. A discussion of the effects of damping on the flutter speed of stressed panels of uniform thickness can be found in Refs. 10 and 11.

## Numerical Results

The gradient projection method of Ref. 7 was used to obtain the solutions presented here. The main computational building block in this optimal control method involves the forward integration of the 8th-order system (2) followed by a backward integration of a 32nd-order system of influence function equations used to enforce satisfaction of the four boundary conditions at $x = 1$. The sum of the squares of these four terminal values is required to be less than $10^{-12}$ at the end of each iteration. The numerical integration is carried out using a 4th-order Runge-Kutta technique with a fixed step size of 0.01. All

**Table 1 Critical aerodynamic parameter values**[8]

| $g_o$ | $\lambda_o$ |
|---|---|
| 0.00 | 343.356 (Refs. 4 and 7) |
| 0.50 | 351 |
| 1.00 | 377 |
| 1.25 | 391 |
| 1.50 | 416 |
| 1.75 | 439 |
| 1.80 | 444 |

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Index category: Structural Design, Optimal.

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**Table 2 Optimal values for the simply-supported solid panel with $R_x = 0$ and $t_{min} = 0.5$**

| $g_o$ | Percent mass reduction | $\alpha$ | $p_R(0)$ | $p_I(0)$ | $r_I(0)$ |
|---|---|---|---|---|---|
| 0.00 | 10.400 | 1.881 | $-3.29(10^{-2})$ | ... | ... |
| 0.50 | 10.621 | 1.900 | $-3.30(10^{-2})$ | $1.73(10^{-2})$ | $5.86(10^{-4})$ |
| 1.00 | 11.216 | 1.928 | $-1.73(10^{-2})$ | $3.41(10^{-2})$ | $1.00(10^{-3})$ |
| 1.25 | 12.316 | 1.947 | $-1.63(10^{-3})$ | $3.47(10^{-2})$ | $1.58(10^{-3})$ |
| 1.50 | 13.342 | 1.982 | $9.93(10^{-3})$ | $2.26(10^{-2})$ | $1.42(10^{-3})$ |
| 1.75 | 16.803 | 2.038 | $8.72(10^{-3})$ | $4.63(10^{-3})$ | $7.86(10^{-4})$ |
| 1.80 | 19.142 | 2.069 | $6.09(10^{-3})$ | $-7.92(10^{-4})$ | $-1.30(10^{-4})$ |

calculations were performed in double precision on an IBM 360/65 computer. Finally, the inequality constraint (4) is enforced using the transformation[7]

$$t(x) = t_{min} + (1/2)u^2(x) \qquad 0 \leqq x \leqq 1 \qquad (5)$$

To the author's knowledge, the only published values of the critical aerodynamic parameter $\lambda_o$ for the uniform reference panel with aerodynamic damping are those given by Houbolt[8] for the case of no inplane loading. Even these values are available only in graphical form. The specific values obtained from Ref. 8 and used in this study are listed in Table 1.

The principal results for the case of $R_x = 0$ and $t_{min} = 0.5$ are collected in Table 2. Note that an additional weight savings approaching 9% is available for sufficiently high damping levels. The optimal frequency parameter and the optimal values for $p_R(0)$, $p_I(0)$, and $r_I(0)$, all of which are treated as control parameters, are also listed in Table 2. Representative optimal thickness and deflection distributions are shown in Figs. 1 and 2, respectively. As is now expected for this class of problems,[3,7] the minimum-weight thickness distribution is symmetrical about the midpoint. The optimal thickness distribution for $g_o = 0.5$ in Fig. 1 is nearly indistinguishable from that for the $g_o = 0$ case.

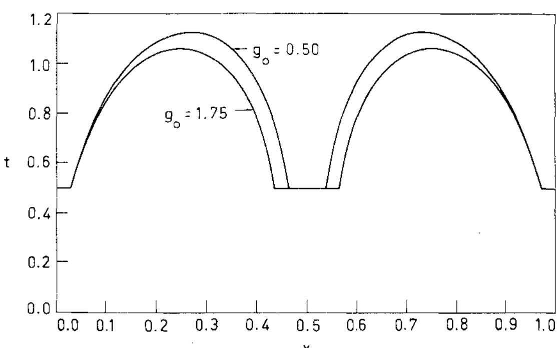


**Fig. 1 Optimal thickness ratio distributions for the simply supported solid panel with $R_x = 0$ and $t_{min} = 0.5$.**

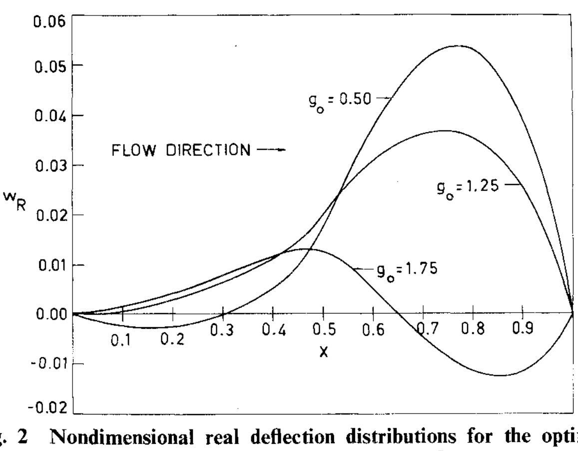


**Fig. 2 Nondimensional real deflection distributions for the optimal simply supported solid panel with $R_x = 0$ and $t_{min} = 0.5$.**

## Conclusions

The minimum panel weight decreases with increasing $g_o$, but for typical† values of $g_o$ (between 0 and 0.5)[12] the additional weight reduction is negligible. However, for large values of the damping parameter, the weight savings do become significant. The shape of the optimal thickness distribution with damping remains relatively unchanged from that obtained without damping.

Since these panel/flutter optimization problems are known to be quite difficult, it was hoped that the addition of physical damping would also serve to reduce the degree of numerical sensitivity inherent in the computational process as well. This beneficial effect was evident only for small values of $g_o$. As $g_o$ was increased above 1.0, the solutions became progressively more difficult to obtain. This is evidently the result of $\lambda_o$ increasing rapidly with increasing $g_o$, since a general observation is that these problems become more sensitive as $\lambda_o$ increases.[7]

It may be of some interest to study the presence of aerodynamic damping for nonzero inplane loading, other boundary conditions, and other minimum thickness constraint levels, but there is little reason to expect any substantial departure from previously established trends. Other damping models may be worth further investigation.

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† As an example, for an aluminum panel ($E = 10^7$ lb/in.$^2$, density = 0.1 lb/in.$^3$, Poisson's ratio = 0.3) at standard sea level conditions, the largest possible damping parameter value is $g_o = 0.285$ and corresponds to a Mach number of 2.30 and a panel length-to-thickness ratio of 182.

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# Stability of Systems with Multiple Motion-Dependent Loading

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## I. Introduction

THE various problems of the stability of elastic bodies subjected to nonconservative single loading have been treated by many authors.[1–10] Problems with multiparameter loading where the applied loads have been assumed to have prescribed motion dependency have also been studied,[11,12] where load magnitudes were allowed to be independent of each other. Problems in which the applied loads were assumed to be dependent on the gradient of the motion, as well as the motion itself, were treated elsewhere.[13]

It appears that a certain aspect of load generalization, not reported in the literature, lies in allowing the applied loads to have independent motion-dependencies. To study the salient features of such a class of problems, a simple two-degree-of-freedom model is considered in this paper. The model is assumed to be subjected to two generally nonconservative forces with independent motion-dependency parameters. The stability investigation is carried out and results in some interesting, and seemingly new, phenomena regarding the stability characteristics. The notion has also been extended to the study of coupled structural systems under this type of loading.[14]

## II. Problem Formulation and Solution

Consider Fig. 1, which is a two-degree-of-freedom model of a column acted upon by two simultaneous subtangential loadings, and in which the values of the concentrated masses and concentrated flexibilities are arbitrarily prescribed. To analyze the stability of this system, we apply the Lagrangian formulation and obtain the following linearized equations of motion

$$\begin{aligned} ml^2(3\ddot{\phi}_1+\ddot{\phi}_2)+C[2-(2-\alpha)Pl]\phi_1+C(-1+\beta\lambda)\phi_2 &= 0 \\ ml^2(\ddot{\phi}_1+\ddot{\phi}_2)-C\phi_1+[1-(1-\beta)l]\phi_2 &= 0 \end{aligned} \qquad (1)$$

Let

$$\phi_i = A_i e^{\Omega t}; \quad i=1,2; \quad \omega^2 = \frac{ml^2}{C}\Omega^2; \quad \lambda = \frac{Pl}{C} \qquad (2)$$

Upon substitution of Eq. (2) into (1), and after appropriate manipulations we obtain the following characteristic equation of the system

$$2\omega^4+[7+(\alpha-2\beta-5)\lambda]\omega^2+[(7-\alpha)(1-\beta)\lambda^2+(\alpha+3\beta-4)\lambda+1]=0 \qquad (3)$$

through the utilization of known arguments on the static and kinetic criteria of stability.[10] With the help of Eq. (3) we deduce the expressions for $\lambda_s$ and $\lambda_k$, the static and kinetic critical loads, respectively. They read

$$\lambda_s = \frac{-(\alpha+3\beta-4)\pm(\Delta_s)^{1/2}}{2(2-\alpha)(1-\beta)} \qquad (4)$$

wherein

$$\Delta_s = \alpha^2+9\beta^2+2\alpha\beta-4\alpha-16\beta+8 \qquad (5)$$

$$\lambda_k = \frac{-(3\alpha+2\beta-19)\pm(\Delta_k)^{1/2}}{(\alpha^2+4\beta^2-4\alpha\beta-2\alpha-4\beta+9)} \qquad (6)$$

where

$$\Delta_k = -32\alpha^2-160\beta^2+176\alpha\beta-32\alpha+88\beta-8 \qquad (7)$$

As to the question of the range of applicability of static and kinetic methods, it is observed that their respective domains of application are determined to a great extent by the signs of $\Delta_s$ and $\Delta_k$ given by Eqs. (5) and (7). Moreover, it is noticed that the inclusion of another parameter of motion dependency has added a new dimension to the space of stability parameters, and hence it has broadened the range of respective stability domains. To investigate this matter in more detail, the plots of $\Delta_s$ and $\Delta_k$, together with their associated signs, are shown in Fig. 2. It appears from the form of the curves, that the comparison of this problem to one with an $n$ motion-dependency parameter would lead to a stability diagram in the $n$-dimensional space in which the curves would be of hyperconic type sections.

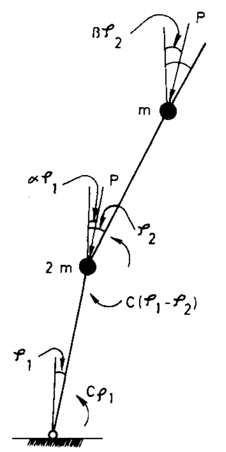


**Fig. 1 Two-degrees-of-freedom model of an elastic system subjected to a multimotion dependent load.**

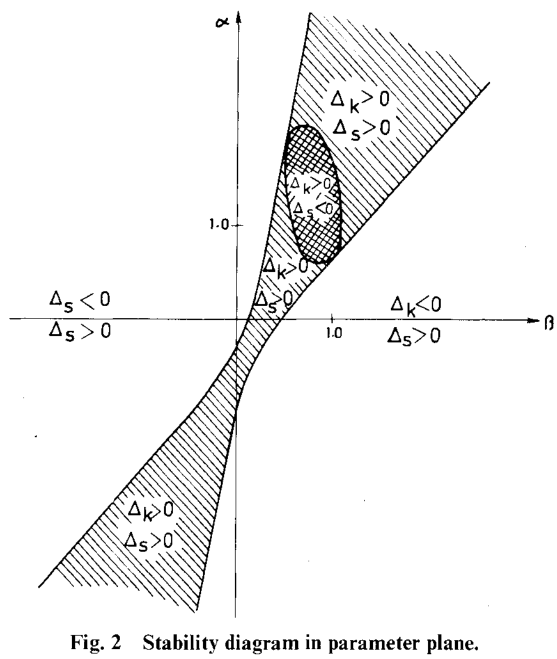


**Fig. 2 Stability diagram in parameter plane.**

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